

Tensor effects on gap evolution of $N = 40$ from non-relativistic and relativistic mean-field theory

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Tensor effects on the $N = 40$ gap evolution of $N = 40$ isotones are studied by employing the Skyrme-Hartree-Fock-Bogoliubov (SHFB) and relativistic Hartree-Fock-Bogoliubov (RHFB) theories. The results with and without the inclusion of the tensor component are compared with the experimental data. When the tensor force is included, both of the two different approaches are found to give the same trend and agree with the experimental one, which indicates the necessity of introducing the tensor force in the evolution of $N = 40$ subshell and on the other hand the reliability of the methods. Furthermore, it is shown that the gap evolution is primarily determined by the corresponding tensor contributions from π and ρ -tensor coupling in the relativistic framework.

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With worldwide development of Radioactive Ion Beam (RIB) facilities, the nuclei far from the valley of β -stability, i.e., the so-called exotic nuclei, have become more accessible in recent years [1]. The investigations on the shell structures of such nuclei have formed a new frontier in modern nuclear physics due to some novel phenomena discovered timely. The typical examples are the emergence of some new magic numbers [2, 3] and the quenching of some conventional shell closures [2, 4] when approaching the isospin limits of the realistic nuclei. As suggested by Otsuka and the collaborators, the tensor force may be one of the crucial physical mechanisms in modeling the shell evolution and the occurrence of magic numbers in the exotic regions [5–7]. However, in both non-relativistic and relativistic self-consistent mean-field calculations, the relevant tensor components were dropped up to very recently [8–13].

Since the magic character of ^{68}Ni was proposed in the early eighties by Bernas *et al.* [14], the shell closure of $N = 40$ has been devoted more and more efforts from both experimental and theoretical sides due to the significant roles played by the nuclei of $N/Z = 40 \sim 60$ involved in the r - and rp -process paths [15]. Although the existence of the $N = 40$ shell gap is well recognized in ^{68}Ni , away from that the uncertainties have remained for many years on the persistence of the shell closures. Until recently several experiments of β -decay indicate a rapidly weakening magicity of $N = 40$ away from ^{68}Ni [15–18], whereas such trend cannot be properly reproduced by shell model and mean-field calculations when the tensor components of the nuclear interaction are excluded [15, 19].

In this Brief Report, the isospin evolutions of subshell $N = 40$, extracted from the self-consistent calculations of $N = 39, 40, 41$ isotones, are studied within the non-relativistic and relativistic mean field theories, specifically the non-relativistic Skyrme-Hartree-Fock-Bogoliubov (SHFB) [20], the relativistic Hartree-

Bogoliubov (RHB) [21, 22], and relativistic Hartree-Fock-Bogoliubov (RHFB) [23] theories. Particular attention is paid to the effects of tensor couplings embedded in the neutron-proton interaction. In both non-relativistic and relativistic cases, the pairing correlations are treated within the Bogoliubov scheme such that the continuum effects can be naturally involved, which is meaningful to provide self-consistent calculations for the selected isotones.

Within the SHFB theory, the tensor interaction is introduced as [20]

$$v_T = \frac{T}{2} \left[\left((\boldsymbol{\sigma}_1 \cdot \mathbf{k}') (\boldsymbol{\sigma}_2 \cdot \mathbf{k}') - \frac{1}{3} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \mathbf{k}'^2 \right) \delta(\mathbf{r}_1 - \mathbf{r}_2) \right] + \frac{T}{2} \delta(\mathbf{r}_1 - \mathbf{r}_2) \left[\left((\boldsymbol{\sigma}_1 \cdot \mathbf{k}) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) - \frac{1}{3} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \mathbf{k}^2 \right) \right] + U \left[(\boldsymbol{\sigma}_1 \cdot \mathbf{k}') \delta(\mathbf{r}_1 - \mathbf{r}_2) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) - \frac{1}{3} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \times (\mathbf{k}' \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{k}) \right], \quad (1)$$

where $\mathbf{k} = (\vec{\nabla}_1 - \vec{\nabla}_2)/(2i)$ acts on the right and $\mathbf{k}' = -(\vec{\nabla}_1 - \vec{\nabla}_2)/(2i)$ acts on the left. The coupling constants T and U denote the strength of the triplet-even and triplet-odd tensor components, respectively. In fact it has been demonstrated that the tensor force [see eq. (1)] plays an important role in determining the evolution of the nuclear shell structure [24, 25].

Notice that the tensor force of Eq. (1) is in a non-relativistic form, inconsistent with the Lorentz covariant principle. Within the relativistic framework, such tensor correlations in $T = 0$ channels can be naturally introduced with the inclusion of the π and ρ -tensor couplings, e.g., in the relativistic Hartree-Fock (RHF) mean field [12, 26]. Within RHFB, the π and ρ -tensor couplings are

adopted as [12]

$$H_\phi = \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \bar{\psi}(\mathbf{r}) \bar{\psi}(\mathbf{r}') \Gamma_\phi(\mathbf{r}, \mathbf{r}') D_\phi(\mathbf{r}, \mathbf{r}') \psi(\mathbf{r}') \psi(\mathbf{r}), \quad (2)$$

where ϕ denote the pseudo-vector π and tensor ρ couplings, $D_\phi(\mathbf{r}, \mathbf{r}')$ are the Yukawa propagators, and the interaction matrices $\Gamma_\phi(\mathbf{r}, \mathbf{r}')$ read as,

$$\Gamma_\pi(\mathbf{r}, \mathbf{r}') \equiv -\frac{1}{m_\pi^2} (f_\pi \vec{\tau} \gamma_5 \gamma_\mu \partial^\mu)_\mathbf{r} \cdot (f_\pi \vec{\tau} \gamma_5 \gamma_\nu \partial^\nu)_{\mathbf{r}'}, \quad (3a)$$

$$\Gamma_\rho^T(\mathbf{r}, \mathbf{r}') \equiv \frac{1}{4M^2} (f_\rho \sigma_{\nu k} \vec{\tau} \partial^k)_\mathbf{r} \cdot (f_\rho \sigma^{\nu l} \vec{\tau} \partial_l)_{\mathbf{r}'}. \quad (3b)$$

In the above equations $M(m_\pi)$ denotes the mass of the nucleon (π meson) and $f_\pi(f_\rho)$ is the coupling constant of $\pi(\rho)$ meson which is exponentially density-dependent [12]. Here what should be clarified is that the tensors expressed above are associated with the Lorentz rotation, different from the quantities defined by the space rotation, e.g., the ones in Eq. (1). In fact it is also illustrated that the π and ρ -tensor couplings, particularly the tensor components reduced with non-relativistic limit, play a crucial role in the self-consistent description of shell structure and corresponding evolution [12, 13, 26].

To clarify the tensor effects in modeling the shell evolution of $N = 40$, different effective interactions are employed in the present work. For the SHFB theory, the effective interaction SLy5 [27] and the one with tensor components added perturbatively (marked by SLy5+T) are adopted [28]. Besides SLy5, another Skyrme force T31 and the one with the tensor force (T31+T) are also utilized for comparison. In the RHFB calculations we choose the effective interactions PKO2, PKO3 and PKA1, comparing to those by RHB with DD-ME2 [29] in which the Fock terms are dropped. It should be mentioned that the effective Lagrangian PKO2 does not contain π or tensor ρ couplings, whereas the former is included in PKO3, and both are taken into account by PKA1.

Aiming at the isospin evolution behavior of sub-shell $N = 40$ close to the neutron drip line, we select six neutron-rich isotones from $Z = 20$ to 30, which are spherical or nearly spherical as predicted by both macroscopic-microscopic model [30] and self-consistent mean-field theory [19]. Experimentally it is hard to measure the sub-shell $N = 40$ directly, especially for the exotic isotones, e.g., ^{60}Ca . As the experimental references, such shell gaps can be approximated by the difference of single-neutron separation energies S_n around the magic or semi-magic neutron number N_m [30, 31]

$$E_{\text{gap}}(Z, N_m) = S_n(Z, N_m) - S_n(Z, N_m + 1) \quad (4)$$

In fact our RHFB calculations also show that the evaluations of sub-shell $N = 40$ by Eq. (4) agree with the energy gaps between the corresponding single-particle levels to certain precisions (~ 200 keV). For convenience we

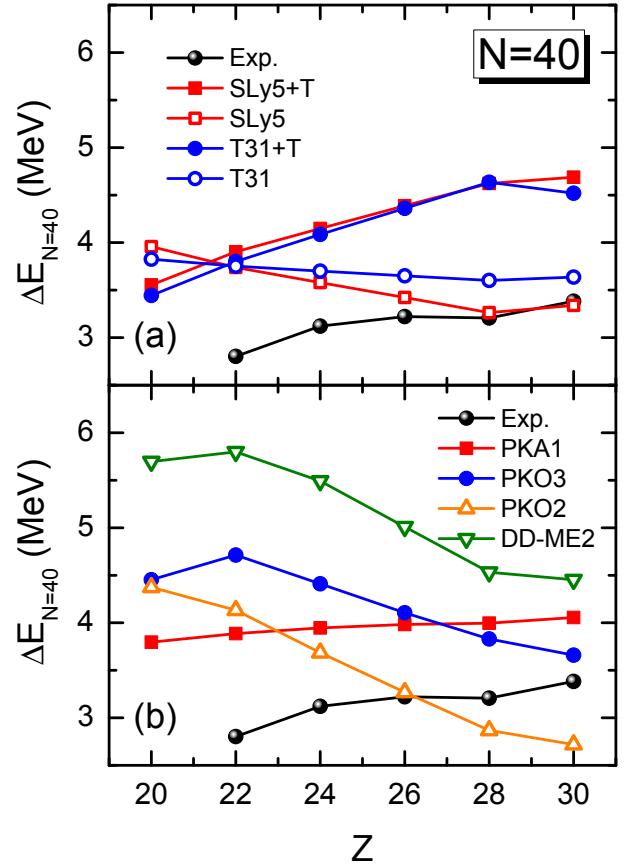


FIG. 1: (Color online) The neutron gap at $N = 40$ of $N = 40$ isotones as functions of the proton number Z . The results are calculated by the non-relativistic (a) and relativistic (b) mean-field theory without and with the tensor component. Experimental values approximated by Eq. (4) are also shown for comparison. See the text for details.

choose the later as the theoretical sub-shells of $N = 40$ in the following discussions.

In Fig. 1 are shown the sub-shell gaps of $N = 40$ along the isotonic chain of $N = 40$, extracted from the calculations of the non-relativistic [plot (a)] and relativistic [plot (b)] mean field theories. As the reference, the experimental data [32] extracted by Eq. (4) are denoted by the black balls. It is found that the experimental E_{gap} is consistent with the β -decay studies by Mülleler *et al.* [17] that the semi-magicity of $N = 40$ becomes weak when moving away from ^{68}Ni . From Fig. 1(a) one can clearly see that the experimental trend cannot be reproduced by the Skyrme forces SLy5 and T31 without tensor components. In contrast, when the tensor force participates in the neutron-proton interaction, identical systematics with the data can be provided by the calculations with SLy5+T and T31+T. Such distinct improvement can be well understood from the nature of tensor force [5]. In the SHFB calculations, the sub-shell of $N = 40$ is determined by the energy gap between the neutron orbits $\nu f_{5/2}$ (or $\nu p_{1/2}$) and $\nu g_{9/2}$, i.e., the ones with $j_{<} = l - 1/2$

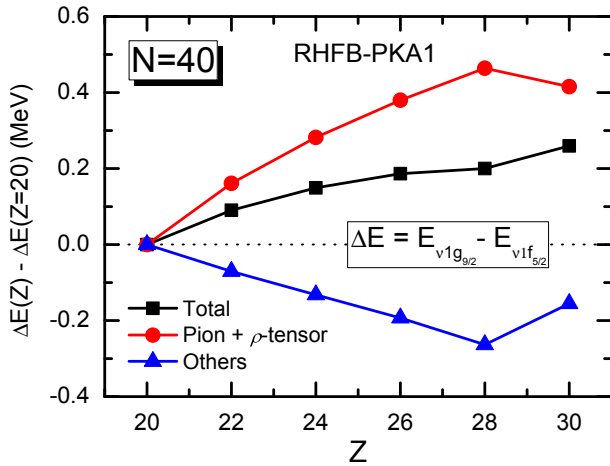


FIG. 2: (Color online) Detailed contributions of the neutron gap at $N = 40$ along $N = 40$ isotones from the isovector π and tensor ρ couplings in comparison with those from the other channels associated with the kinetic energy, σ -scalar, ω -vector, ρ -vector couplings, and the rearrangement term. The results are extracted from the RHFb calculations with PKA1.

and $j_> = l + 1/2$, respectively. Along the isotonic chain from ^{60}Ca to ^{68}Ni , the valence protons will gradually occupy the proton orbit $\pi 1f_{7/2}$ with $j_> = l + 1/2$, which presents repulsive tensor couplings with the orbit $\nu g_{9/2}$ ($j_> = l + 1/2$) and attractive ones with $\nu f_{5/2}$ (or $\nu p_{1/2}$). As a result the semi-magicity of $N = 40$ are remarkably weakened by the tensor force when moving away from ^{68}Ni , in accordance with the experimental reference data [32] as well as the β -decay studies [17]. As compared to the SHFB calculations excluding tensor components it is then well demonstrated for the crucial role played by the tensor force in modeling the shell evolution [25, 28].

Similar systematical agreements and discrepancies are also found between the relativistic calculations and the experimental data. As seen from Fig. 1(b) the calculations of RHB with DD-ME2 and RHFb with PKO2 present opposite isospin evolution behavior to the experimental trend, in which the isovector tensor component π -coupling is not taken into account. In fact due to the lack of isovector tensor couplings similar inconsistencies are also found in other RHB calculations, e.g., with PKDD [33]. Compared to those excluding isovector tensor components, the RHFb calculations with PKA1 present identical trend as the data, which is mainly due to the effects of tensor components embedded in isovector π and tensor ρ couplings, particularly the former one. As shown in Fig. 2, only the contributions from π and tensor ρ -couplings are consistent with the experimental trend while the other channels present fairly remarkable cancellation with the referred behaviors. Consequently the calculations with PKA1 lead to fairly weak isospin dependence on the shell evolution when comparing with the non-relativistic results and the experimental trend. In fact the π contributions extracted from the calcula-

tions with PKO3 also present consistent trend with the experimental data (see Fig. 3) while too strong cancellations are contributed by the other channels, leading to opposite isospin evolution in total [see Fig. 1 (b)].

It is interesting that the non-relativistic SHFB calculations with the tensor force and the relativistic RHFb ones including the π - and ρ -tensor couplings favor the same trend. Different models may validate each other, which to a large extent indicates the reliability of the theoretical models and thus one can obtain more compelling results. In addition, recent experimental [34, 35] and theoretical [36] studies seem to indicate that a quantum phase transition from the spherical to deformed shapes takes place near $N = 40$ for Cr and Fe isotopes, referred to as a “new island of inversion”. Similar to the one of $N = 20$ [37–39], the island of inversion of $N = 40$ in the neutron-rich region, which can only be described appropriately with the inclusion of the tensor force as one can see from Fig. 1 and 2.

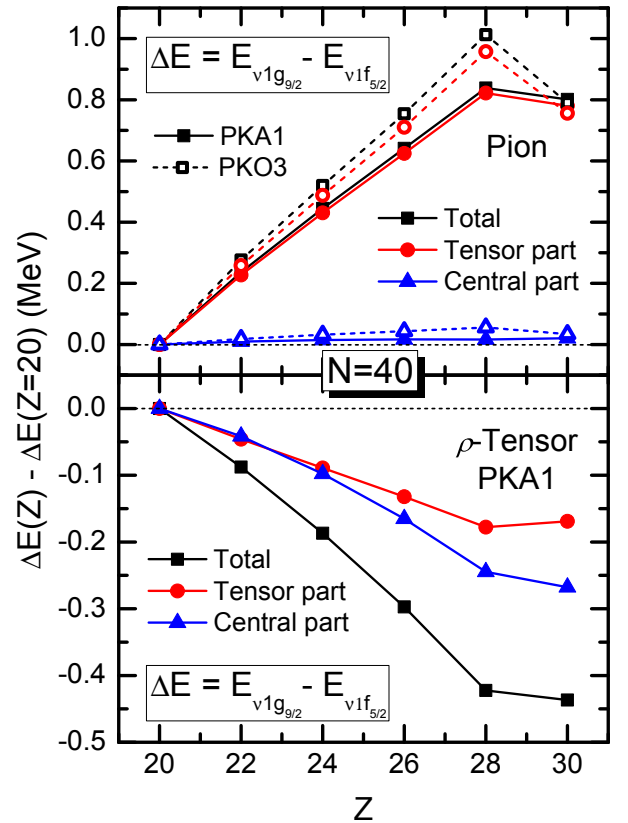


FIG. 3: (Color online) Detailed contributions from the tensor and central parts of pion (upper panel) and ρ -tensor (lower panel) couplings to the energy difference $\Delta E = E_{\nu 1g_{9/2}} - E_{\nu 1f_{5/2}}$ between the neutron canonical single-particle states $\nu 1g_{9/2}$ and $\nu 1f_{5/2}$ along the isotonic chain of $N = 40$. The results are extracted from the RHFb calculations with PKA1 (filled symbols) and PKO3 (open symbols).

To further clarify the contributions from the tensor coupling in RHFb theory, we shown in Fig. 3 the contri-

butions from the tensor and central parts of pion (upper panel) and ρ -tensor (lower panel) couplings to the evolutions of the energy difference $\Delta E_{gf} = E_{\nu 1g_{9/2}} - E_{\nu 1f_{5/2}}$, which corresponds to the $N = 40$ shell gap in PKA1 calculations along the isotonic chain. Here the tensor part of ρ -tensor coupling is obtained from similar non-relativistic reduction as the π pseudo-vector coupling [26, 40]. For the isovector π coupling which in total presents consistent results with the data (see Fig. 1), its tensor part dominates the isospin dependence whereas the central part presents nearly constant isospin evolution in both PKA1 and PKO3 calculations. On the contrary, both the tensor and central parts of ρ -tensor coupling provide almost equivalent contributions to the isospin evolution of ΔE_{gf} , both are opposite with the π effects. It is also due to such remarkable cancellation between the π and ρ -tensor couplings that PKA1 provides consistent while fairly weak isospin-dependent behavior as referred to the data.

In summary, the tensor effects in modeling the semimagicity of $N = 40$ are studied within the non-relativistic

Skyrme-Hartree-Fock-Bogoliubov (SHFB) theory and the relativistic Hartree-Fock-Bogoliubov (RHFB) theory. The calculations with and without isovector tensor component compared with the experimental data indicate that the tensor forces play a crucial role in reproducing the isospin evolution of the $N = 40$ subshell. It is also interesting that, when the tensor force is included, both present non-relativistic and relativistic methods yield the same trend for this shell gap evaluation. These two different models validate each other and lead to a more convincing results. In the RHFB calculations, both π -tensor and ρ -tensor couplings present significant contributions in the $T = 0$ NN interaction, which eventually determine the gap evolution of the $N = 40$ subshell, in agreement with the experimental trend.

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